Business Calculus Test 2 Review Answers

Dr. Graham-Squire, Spring 2017

1. Find $\dfrac{dy}{dx}$ for the equation $x^3 + xy^2 + y^3 = 0$.

   Ans: $\dfrac{dy}{dx} = \dfrac{-3x^2 - y^2}{2xy + 3y^2}$

2. Dominic has attached his sister Eva to a kite and is letting her drift away in the wind. Assuming that the kite stays at a constant height of 100 feet above the ground and kite string is coming out of the spool at a constant rate of $5\sqrt{3}$ feet/minute, find the rate at which the horizontal distance between Dominic and Eva (that is, the distance between Dominic and the point directly beneath Eva on the ground) is changing when the kite string is 200 feet long.

   Ans: 10

3. For each function, find
   (a) the intervals where the function is increasing or decreasing,
   (b) any relative maximum or minimum points (if any),
   (c) the intervals where f is concave up or down, and
   (d) inflection points (if any). For fun, you can also
   (e) sketch a graph of the function from the information you found, then compare to what you get when you put it into a graphing calculator.

   (i) $f(x) = x^4 - 2x^2$
   (ii) $f(x) = x\sqrt{x-1}$

   Ans: (i) is increasing on (-1,0) and (1,∞), decreasing on $(-\infty, -1)$ and $(0, 1)$. minimums at $(\pm 1, -1)$ and a max at (0,0). It is concave up on $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$ and concave down on $(-1/\sqrt{3}, 1/\sqrt{3})$, with inflection points at $(\pm 1/\sqrt{3}, -1/9)$.

   (ii) is always increasing, has no max or min. Concave down on $(1, 8/3)$ and concave up on $(8/3, \infty)$, with inflection point at $(8/3, 8\sqrt{5}/3\sqrt{3})$.

4. Find the absolute maximum and minimum (if they exist) of the function $g(x) = x\sqrt{4 - x^2}$ on the interval [0,2].

   Ans: Absolute min of 0 at both $x = 0$ and $x = 2$, abs. max of 2 at $x = \sqrt{2}$.

5. A rectangular box is to have a square base and a volume of 20 ft^3. If the material for the base costs 30 cents/ft^2, the material for the top costs 20 cents/ft^2, and the material for the sides costs 20 cents/ft^2, determine the dimensions of the box that give a minimum cost. Check your answer to make sure it is a minimum.

   Ans: 2.52 ft by 2.52 ft by 3.15 ft (where the 3.15 is the height). You also need to use the first or second derivative test to confirm that your answer is a minimum.

6. The number of internet users in China is approximated by the function

$$N(t) = 94.5e^{0.2t} \qquad\qquad (1 \le t \le 6)$$

where $N(t)$ is measured in millions and t is years with $t = 1$ being 2005.

(a) How many users are there in 2010?

Ans: 313,751,049 users.

(b) When did the number of users equal 190,300,000?

Ans: When $t = 3.5$, so approximately the middle of 2007.

7. Expand and simplify the expression $\ln \dfrac{x^2 \cdot e^{3x}}{\sqrt{x}(1+x)^2}$.

Ans: $1.5 \ln x + 3x - 2\ln(1+x)$

8. Find the interest rate needed for an investment of \$4000 to double in 5 years if interest is compounded continuously.

Ans: 13.86%

9. Find $f'(x)$ if $f(x) = \ln \dfrac{e^{3x}+4}{8}$.

Ans: $f'(x) = \dfrac{3e^{3x}}{e^{3x}+4}$.

10. The percentage of alcohol in a person's bloodstream t hr after drinking 8 fluid oz of whiskey is given by

$$A(t) = 0.23te^{-0.4t}$$

(a) How fast is the percentage changing after 1 hour? 0.0925

After 4 hours? -0.0279

(b) Use calculus to find at what value of t is the percentage at a *maximum*. When $t = 2.5$.

What is the percentage at that time? 0.21 (Way above the legal limit of 0.08).

11. Use logarithmic differentiation to find $f'(x)$ if $f(x) = x^{2x}$.

Ans: $f'(x) = x^{2x}(2\ln x + 2)$.

12. The element Grahamsquireium has a half-life of 250 years. Given a 100 gram sample, how much of it will be left after 300 years?

Ans: 43.53 grams.

13. Find the indefinite integral $\displaystyle\int x\left(\sqrt{x} + \frac{3}{x^2} - \frac{2e^x}{x}\right) dx$.

Ans: $\frac{2}{5}x^{5/2} + 3\ln|x| - 2e^x + C$ (Note: $\ln x$ is also correct)

14. Find the indefinite integrals:

(a) $\displaystyle\int x^2(2x^3+3)^4\,dx$.

Ans: $\frac{1}{30}(2x^3+3)^5 + C$. Let $u = 2x^3+3$.

(b) $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx$. Let $u = \ln x$.

Ans: $-(\ln x)^{-1} + C$